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An analysis of the inclusive $\Upsilon(1S) \to \eta' X$ decay and constraints on the η'-meson distribution amplitudes*

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Abstract. We calculate the η'-meson energy spectrum in the $\Upsilon(1S) \to \eta' ggg \to \eta' X$ decay in the leading-order perturbative QCD in the static-quark limit for the orthoquarkonium. Our principal result is the extraction of parameters of the $\eta' g^* g$ effective vertex function (EVF) involving a virtual and a real gluon from the available data on the hard part of the η'-meson energy spectrum. The perturbative-QCD based framework provides a good description of the available CLEO data, allowing one to constrain the lowest Gegenbauer coefficients $B_2^{(q)}$ and $B_2^{(g)}$ of the quark-antiquark and gluonic distribution amplitudes of the η'-meson. The resulting constraints are combined with the existing ones on these coefficients from an analysis of the $\eta' - \gamma$ transition form factor and the requirement of positivity of the EVF, yielding $B_2^{(q)}(\mu_0^2) = -0.008 \pm 0.054$ and $B_2^{(g)}(\mu_0^2) = 4.6 \pm 2.5$ for $\mu_0^2 = 2$ GeV^2. This reduces significantly the current uncertainty on these coefficients.

1 Introduction

A quantitative description of the rare decays with the η'-meson production, such as $B \to \eta' K^{(*)}$, $B \to \eta' X_s$, $\Upsilon(1S) \to \eta' X$, requires an understanding of the $\eta' g^* g^{(*)}$ effective vertex function (EVF), $F_{\eta' g^* g^{(*)}}(q_1^2, q_2^2, m_{\eta'}^2)$ [also called the $\eta' - g$ transition form factor when one of the gluons is on the mass shell]. If one or both of the gluons entering into the EVF in such decays are far from their mass shell, the dependence of the EVF on the gluon virtualities should be included in the theoretical analysis of decays. For energetic η'-meson, the QCD hard-scattering approach can be used to derive the required EVF [1,2, 3,4]. As the η'-meson has a relatively large mass, $m_{\eta'} = 958$ MeV, its effect should be taken into account in applications of the $\eta' g^* g^{(*)}$ EVF to physical processes, in particular when the gluon virtualities are time-like [4]. Moreover, the inclusion of the η'-meson mass results in a pole-like form for the $\eta' - g$ transition form factor:

$$F_{\eta' g}(p^2) \equiv F_{\eta' g^* g}(p^2, 0, m_{\eta'}^2) = \frac{m_{\eta'}^2 \, H(p^2)}{p^2 - m_{\eta'}^2}, \qquad (1)$$

This form was introduced by Kagan and Petrov [5]. These authors also suggested to ignore the dependence of the function $H(p^2)$ on the gluon virtuality and approximate it by a constant value, $H_0 = 1.7$ GeV^{-1}, resulting from the analysis of the $J/\psi \to \eta' \gamma$ decay [6]. Also it was subsequently shown that the hard part of the η'-meson energy spectrum in the inclusive $\Upsilon(1S) \to \eta' X$ decay [7] is in a qualitative agreement with the spectrum measured recently by the CLEO collaboration [8]. Starting form the above observations, a quantitative analysis of the η'-meson energy spectrum in this decay was undertaken by us [9] and its results are briefly summarized in this report.

2 The η'-meson wave-function

In the quark-mixing scheme, the Fock-state decomposition of the η'-meson wave-function is as follows:

$$|\eta'\rangle = \sin\phi \, |\eta'_q\rangle + \cos\phi \, |\eta'_s\rangle + |\eta'_g\rangle, \qquad (2)$$

where $|\eta'_q\rangle \sim |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$, $|\eta'_s\rangle \sim |\bar{s}s\rangle$, and $|\eta'_g\rangle \sim |gg\rangle$ are the light-quark, strange-quark and gluonic components, respectively. For an energetic η'-meson in a process, its wave-function can be described in terms of the quark-antiquark, $\phi_{\eta'}^{(q)}(x, Q^2)$, and gluonic, $\phi_{\eta'}^{(g)}(x, Q^2)$, light-cone distribution amplitudes (LCDAs). In the LCDAs above, x is the momentum fraction of one of the partons inside the meson and Q^2 is a typical hard scale of the process. Note that these LCDAs mix under the scale evolution.

In most applications, approximate forms for the η'-meson LCDAs are usually employed in which only the

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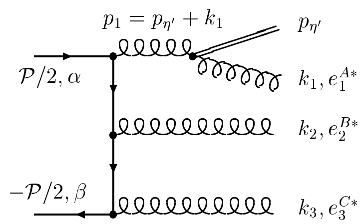

Fig. 1. A typical Feynman diagram describing the $\Upsilon(1S) \to ggg^*(g^* \to \eta' g) \to \eta' X$ decay

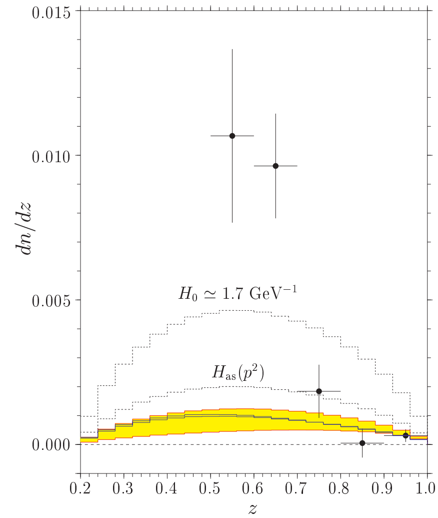

Fig. 2. The η'-meson energy spectrum in the $\Upsilon(1S) \to \eta' X$ decay

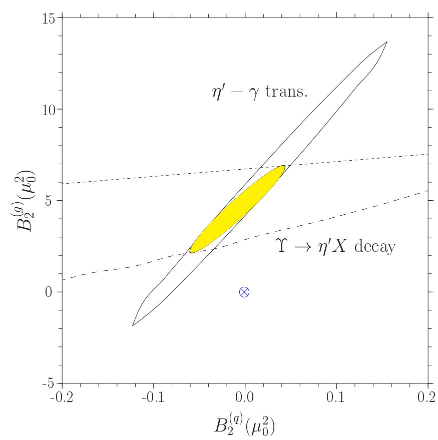

Fig. 3. The resulting 1σ contour (combined best fit), shown by the yellow (shaded) region, for the Gegenbauer coefficients estimated at the scale $\mu_0^2 = 2$ GeV^2 from the data on the $\eta' - \gamma$ transition form factor (solid curve) and $\Upsilon(1S) \to \eta' X$ decay (long-dashed and short-dashed curves)

first non-asymptotic term in both the quark-antiquark and gluonic component is kept:

$$\phi_{\eta'}^{(q)}(x, Q^2) = 6x\bar{x}\left[1 + 6(1 - 5x\bar{x})\, A_2(Q^2) + \ldots\right], \quad (3)$$
$$\phi_{\eta'}^{(g)}(x, Q^2) = 5x^2\bar{x}^2\,(x - \bar{x})\, B_2(Q^2) + \ldots\ .$$

These LCDAs involve the Gegenbauer moments for which the following notation is used:

$$A_2(Q^2) = B_2^{(q)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^2} + \rho_2^{(g)} B_2^{(g)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^2}, \quad (4)$$

$$B_2(Q^2) = \rho_2^{(q)} B_2^{(q)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^2} + B_2^{(g)}\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^2}. \quad (5)$$

The quantities γ_+^2, γ_-^2, $\rho_2^{(q)}$, and $\rho_2^{(g)}$ are determined by the perturbative QCD while the Gegenbauer coefficients $B_2^{(q)}(\mu_0^2)$ and $B_2^{(g)}(\mu_0^2)$ are non-perturbative parameters. These coefficients have to be modeled or extracted from a phenomenological analysis of experimental data.

The first attempt to estimate the Gegenbauer coefficients $B_2^{(q)}(\mu_0^2)$ and $B_2^{(g)}(\mu_0^2)$ was recently undertaken by Kroll and Passek-Kumericki [3]. They performed a NLO theoretical analysis of the $\eta' - \gamma$ transition form factor and extracted the following values:

$$B_2^{(q)}(\mu_0^2 = 1\ \text{GeV}^2) = 0.02 \pm 0.17, \quad (6)$$
$$B_2^{(g)}(\mu_0^2 = 1\ \text{GeV}^2) = 9.0 \pm 11.5,$$

from the CLEO [10] and L3 [11] data. This fit leaves an order of magnitude uncertainty on the coefficients.

The inclusive $\Upsilon(1S) \to \eta' X$ decay allows also to get an additional information on the Gegenbauer coefficients.

3 Perturbative-QCD analysis of $\Upsilon(1S) \to \eta' X$ and comparison with data

One of the 18 diagrams describing the decay $\Upsilon(1S) \to ggg^*(g^* \to \eta' g) \to \eta' X$ in the leading order is presented in Fig. 1; the other 17 diagrams can be obtained from the above one by the permutations of the gluons in the intermediate (virtual) and final states. The static limit for the heavy quark and antiquark in the orthoquarkonium $\Upsilon(1S)$ state is used in the calculations. The total decay amplitude $\mathcal{M}[\Upsilon \to \eta' ggg]$ is rather lengthy and can be found in [9].

The η'-meson energy spectrum can be theoretically determined as follows:

$$\frac{dn}{dz} = \frac{1}{\Gamma_{3g}^{(0)}}\,\frac{1}{3!}\,\frac{1}{(2\pi)^8}\,\frac{1}{2M}\int \frac{d\mathbf{k}_1}{2\omega_1}\,\frac{d\mathbf{k}_2}{2\omega_2}\,\frac{d\mathbf{k}_3}{2\omega_3}\,\frac{d\mathbf{p}_{\eta'}}{2E_{\eta'}} \quad (7)$$
$$\times\, \delta^{(4)}(\mathcal{P} - k_1 - k_2 - k_3 - p_{\eta'})\, \delta(z - 2E_{\eta'}/M)$$
$$\times\, \frac{1}{3}\sum |\mathcal{M}[\Upsilon \to \eta' ggg]|^2\,,$$

where $\Gamma_{3g}^{(0)}$ is the three-gluon decay width of the $\Upsilon(1S)$-meson in the leading order:

$$\Gamma_{3g}^{(0)} = \frac{16}{9}\,(\pi^2 - 9)\, C_F\, B_F\, \alpha_s^3(\mu_\Upsilon^2)\, \frac{|\psi(0)|^2}{M^2}. \quad (8)$$

Here, $C_F = (N_c^2 - 1)/(2N_c)$, $B_F = (N_c^2 - 4)/(2N_c)$, and $\mu_\Upsilon \sim M$ is a typical hard scale of the process.

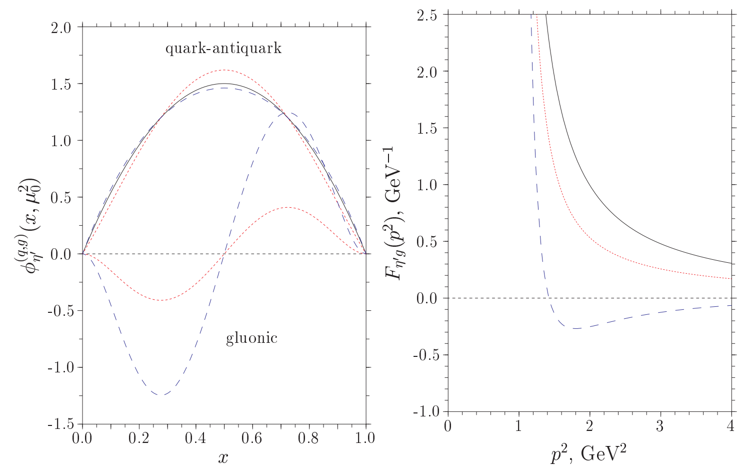

Fig. 4. The η'-meson quark-antiquark $\phi^{(q)}_{\eta'}(x,\mu_0^2)$ and gluonic $\phi^{(g)}_{\eta'}(x,\mu_0^2)$ LCDAs as functions of x (left frame), and the resulting $\eta' - g$ transition form factor (right frame). The solid curves correspond to the asymptotic quark-antiquark LCDA, while the LCDAs for the central values from the combined best-fit region of the Gegenbauer coefficients given in (11) are drawn as dotted curves. The LCDAs with the values $B_2^{(q)} = 0.15$ and $B_2^{(g)} = 13.5$, which are allowed within 1σ from the analysis of the data on the $\eta' - \gamma$ transition form factor [3], are presented as the dashed curves. Note that for this case the function $F_{\eta' g}(p^2)$ is no longer positive definite, as shown in the right frame

As the low-z data are dominated by the fragmentation of gluons into the η'-meson, following the CLEO analysis [8], we concentrate on the last three ($z \geq 0.7$) and four ($z \geq 0.6$) experimental bins (see Fig. 2).

The fit of the $B_2^{(q)}(\mu_0^2)$ and $B_2^{(g)}(\mu_0^2)$ parameters based on the last four experimental data points results in unacceptably large values of the minimum χ^2 [9]. Thus, only the data in the last three bins with $z \geq 0.7$ are used in the analysis (quoted χ^2 corresponds to three degrees of freedom) and yield the following best fits ($\mu_0^2 = 2$ GeV^2):

$$B_2^{(q)}(\mu_0^2) = -0.89^{+1.32}_{-1.58}, \quad B_2^{(g)}(\mu_0^2) = -2.86^{+20.04}_{-5.80}, \quad (9)$$

$$B_2^{(q)}(\mu_0^2) = -1.09^{+1.51}_{-1.36}, \quad B_2^{(g)}(\mu_0^2) = 11.53^{+5.55}_{-20.09}, \quad (10)$$

with $\chi^2 = 2.45$ and 2.37 for each set, respectively.

The overlapping region in the $[B_2^{(q)}(\mu_0^2), B_2^{(g)}(\mu_0^2)]$ parameter space from the $\eta' - \gamma$ transition form factor [3] and the $\Upsilon(1S) \to \eta' X$ decay [9] is presented in Fig. 3. The parameter space from the $\Upsilon(1S) \to \eta' X$ decay fit is obtained by imposing the additional condition that $F_{\eta' g}(p^2)$ remains the positive-definite function for $p^2 > m^2_{\eta'}$ [9] which is illustrated in Fig. 4 in term of both the LCDAs and the $\eta' - g$ transition form factor. The resulting combined best fit of the Gegenbauer coefficients yields [9]:

$$B_2^{(q)}(\mu_0^2) = -0.008 \pm 0.054, \quad B_2^{(g)}(\mu_0^2) = 4.6 \pm 2.5. \quad (11)$$

Using instead the asymptotic LCDA yields $\chi^2 = 8.41$; hence, the asymptotic LCDA is not favored by the current analysis. However, most of this χ^2 is contributed by a single experimental point (see Fig. 2). As data on the hard part of the η'-meson energy spectrum are rather sparse, one can not exclude the asymptotic LCDA based on these data. Hopefully, experimental measurements will be improved soon to draw more quantitative conclusions.

4 Summary

The η'-meson energy spectrum in the $\Upsilon(1S) \to \eta' ggg \to \eta' X$ decay is calculated in the leading-order perturbative QCD in the static-quark limit for the $\Upsilon(1S)$-meson. The leading-twist (twist-two) quark-antiquark and gluonic LCDAs are used to describe the η'-meson wave-function. In the LCDAs, the asymptotic and the first non-asymptotic terms are taken into account. An essential dependence of the energy spectrum on the Gegenbauer coefficients is observed. These Gegenbauer coefficients are determined in the large-z region ($z \geq 0.7$) of the η'-meson energy spectrum from the recent CLEO data, however, the resulting 1σ contour have a large dispersion. Combining this analysis with the one of the $\eta' - \gamma$ transition form factor and requiring additionally that the EVF, $F_{\eta' g}(p^2)$, remains positive-definite in the entire $p^2 > m^2_{\eta'}$ region, yield much improved determination of the Gegenbauer coefficients.

It remains to be seen if the so-determined $\eta' g^* g^*$ EVF explains the data on the inclusive $B \to \eta' X_s$ decay.

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